SAPPHIRE CANTILEVER SPRING CONSTANT

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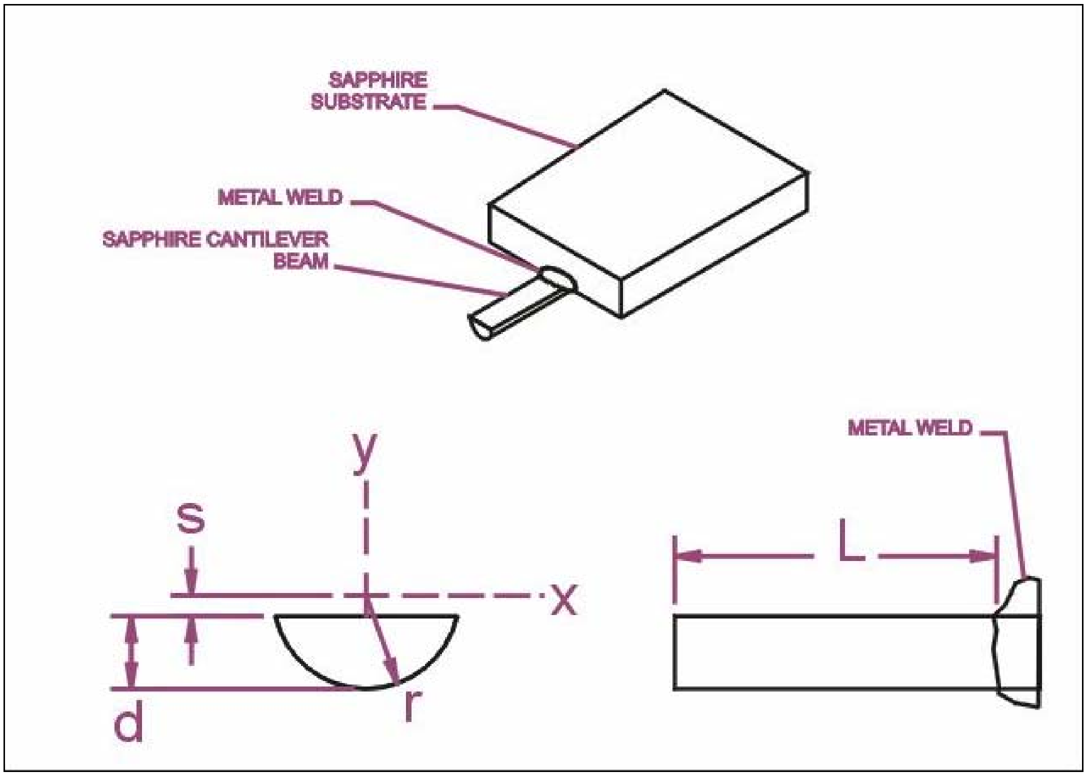
Procedure to make a sapphire cantilever with a given Spring Constant **K.**



Variables

L	Length*	m	
d	Thickness*	m	
r	Beam Radius*	m	$r = 31 \times 10^{-6}$m on all beams
F	Resonant Frequency*	Hz	
δ	Density*	Kg/m^3	$\delta = 3960\ Kg/m^3$ for sapphire
K	Spring Constant	N/m	
I	Beam Section Moment of Inertia	m^4	
E	Young Modulus of Elasticity	Pa	
A	Beam Section Area	m^2	
M	Beam Mass	Kg	

*Measured variables.

Equations

(1) K $= \dfrac{3EI}{L^3}$ N/m

(2) I $= \int_{-r}^{s} 2y^2\sqrt{r^2 - y^2}\,dy$ m^4

(3) A $= \dfrac{\pi \times r^2}{2} + \int_{o}^{s} 2\sqrt{r^2 - y^2}\,dy$ m^2

(4) M_1 $= A \times L_1 \times \delta$ Kg

(5) E $= \dfrac{(2\pi F_1)^2 \times 0.23 M_1 \times L_1^3}{3I}$ Pa

(6) L_2 $= \sqrt[3]{\dfrac{3EI}{K}}$ m

(7) M_2 $= A \times L_2 \times \delta$ Kg

(8) K $= (2\pi F_2)^2 \times 0.23 M_2$ N/m

(9) F_{2C} $= \dfrac{1}{2\pi}\sqrt{\dfrac{3EI}{0.23 M_2 L_2^3}}$ Hz

Procedure

A) A cantilever beam is made with the desired thickness **d** and a length $\mathbf{L_1}$ *longer* than the length predicted by the Cantilever Parameter Selection Graph.

B) The resonant frequency $\mathbf{F_1}$ is measured.

C) **E** is calculated with equation (5).

D) The beam is cut to a new length $\mathbf{L_2}$ using the desired **K** in equation (6).

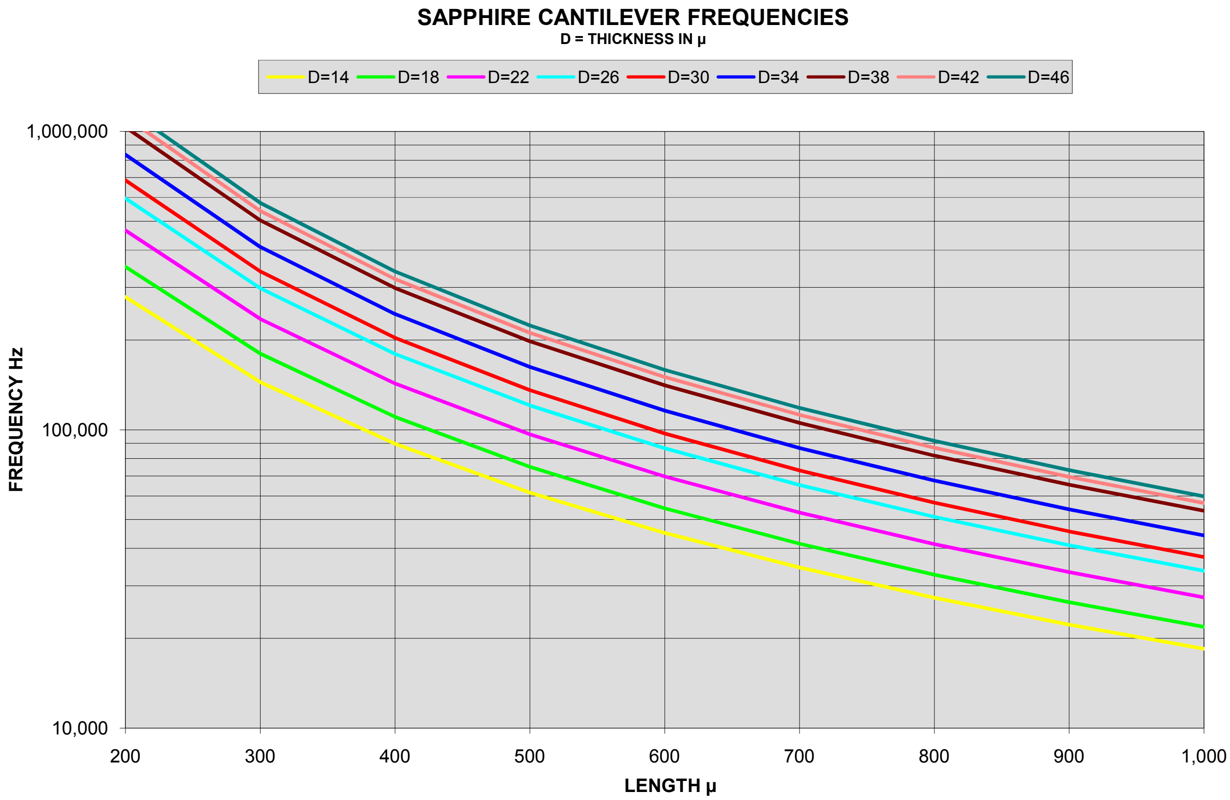
E) A new resonant frequency $\mathbf{F_2}$ is measured and **K** is verified with equation (8).

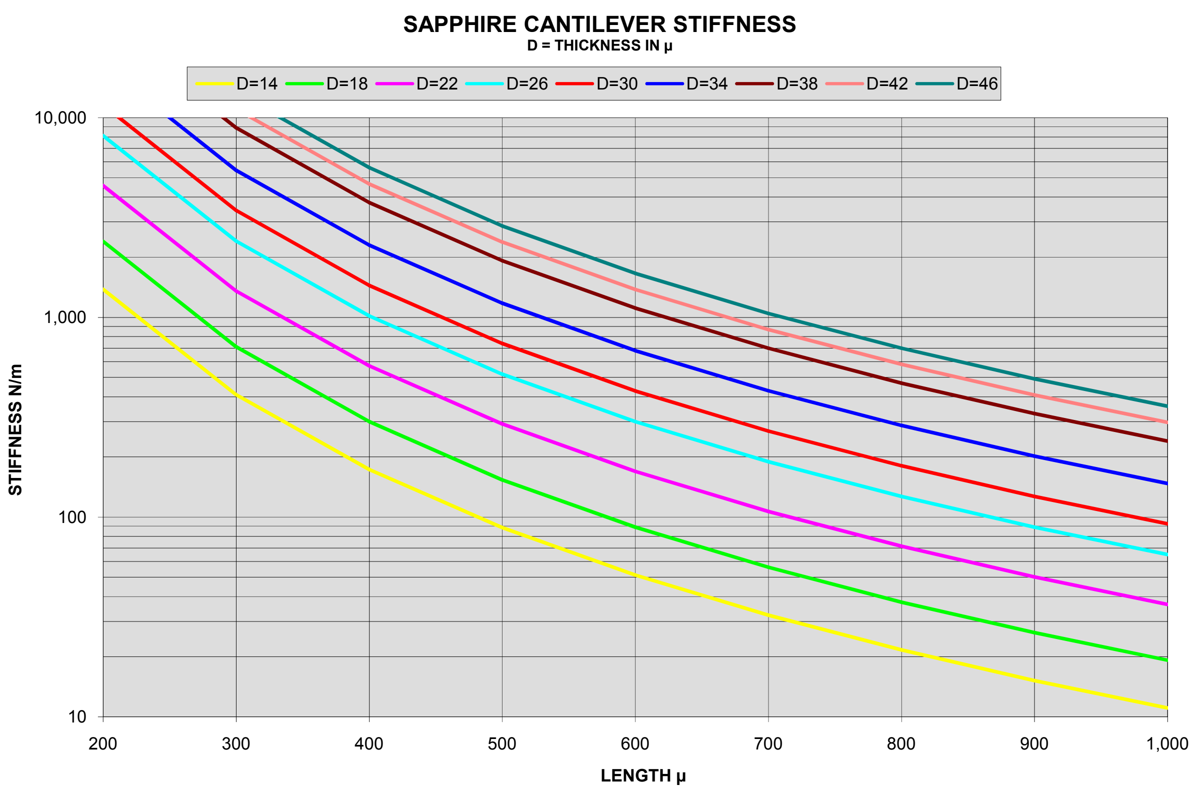
F) The procedure and calculations are corroborated with equation (9) which calculates the expected resonant frequency value $\mathbf{F_{2C}}$ using $\mathbf{L_2}$. Normally the calculated frequency $\mathbf{F_{2C}}$ and the measured frequency $\mathbf{F_2}$ differ less than 1%.

The procedure above applies before a diamond tip is attached to the sapphire cantilever. Adding the tip mass lowers the cantilever resonant frequency but the spring constant remains the same. The graph shows the cantilevers resonant frequencies measured after the tip is attached.

MICRO STAR AFM DIAMOND-SAPPHIRE PROBES

CANTILEVER PARAMETER SELECTION GRAPHS *





* Actual frequencies may vary as much as 30% of values in the graph because the diamond tip weight is estimated.